DISCRETE GEOMETRY DAYS

June 21-24, 2016, Budapest

MŰEGYETEM 1782

SCHEDULE

All talks are held at the Math. Inst. of the Budapest University of Technology and Economics /Egry József u. 1., Budapest/, Building H, Room 607 (6th floor)

Tuesday, June 21

9:00–9:30	*Registration*
9:30–9:45	*Opening by Miklós Horváth (Head of Math. Inst., BUTE)*
9:45–10:25	**Z. Füredi**, *Subtended Angles.*
Coffee break	
10:50–11:30	**T. Bisztriczky**, *On neighbourly 4-polytopes.*
11:40–12:00	**K. Jochemko**, *Unimodality of h^*-vectors for zonotopes and refined Eulerian polynomials: geometry and combinatorics.*
Lunch break	
14:00–14:40	**D. Oliveros**, *Symmetric Snarks.*
14:50–15:10	**D. Pálvölgyi**, *Polychromatic coloring and cover-decomposition problems in the plane.*
Coffee break	
15:40–16:00	**V. Vígh**, *On the angle sum of lines.*
16:05–16:25	**M. Balko**, *On the Beer index of convexity and its variants.*
16:30–16:50	**A. Joós**, *Finding equal-diameter tetrahedralizations of polyhedra.*
Short Coffee break	
17:05–17:25	**R. Kozma**, *The best horoball packings for hyperbolic 3- and 4-space.*
17:30–17:50	**J. Szirmai**, *Hyp-hor packings in n-dimensional hyperbolic spaces.*

Wednesday, June 22

9:00–9:40	**M. Henk**, *Discrete Slicing Problems.*
9:55–10:15	**A. Akopyan**, *On length of curves passing through boundary points of a planar convex shape.*
10:20–10:40	**B. González Merino**, *On the missing boundary of the (r, D, R)-diagram.*
Coffee break	
11:10-11:50	**B. Csikós**, *Monotonicity of the perimeter of the convex hull and the area of the intersection of disks in curved planes.*
12:00–12:20	**G. Ambrus**, *Vector sum estimates in normed spaces.*
Lunch break	
14:00–14:40	**I. Bárány**, *Random spherical polytopes in a halfsphere.*
Coffee break	
15:10–15:30	**I. Talata**, *Examples of convex polytopes with odd translative kissing numbers.*
15:35–15:55	**T. Kobos**, *The Banach-Mazur distance in non-asymptotic setting.*
19:00–	*Conference Dinner:* The Kőleves Garden /address: Kazinczy utca 41./

Thursday, June 23

9:00–9:40 **L. Montejano**, *Rotors in triangles and tetrahedra.*

9:55–10:15 **M. Musielak**, *Reduced spherical convex bodies.*

10:20–10:40 **A. Polyanskii**, *On reduced polytopes.*

Coffee break

11:10-11:50 **K. Böröczky**, *Valuations on lattice polytopes.*

Lunch break

14:00–14:40 **A. Holmsen**, *The intersection of a matroid and an oriented matroid.*

14:50–15:10 **S. Berg**, *Lattice points in centered convex bodies.*

Coffee break

15:40–16:00 **A. Magazinov**, *Half-space depth of the centerline in* $\mathbb{R}^d$.

16:05–16:25 **M. Henze**, *Tight bounds on discrete quantitative Helly numbers.*

16:30–16:50 **L. Silverstein**, *Tensor Valuations on Lattice Polytopes.*

Short Coffee break

17:10–17:30 **F. Kovács**, *Some aspects of orthogonal weavings on convex polyhedra.*

17:35–17:55 **I. Kovács**, *Dense point sets with many halving lines.*

Friday, June 24

9:00–9:40 **J. Solymosi**, *Algebraic Methods in Discrete Geometry.*

9:55–10:15 **E. Makai, Jr.**, *Densest packings of parallel strings and layers of balls.*

10:20–10:40 **H. Lin**, *Sets with few ordinary circles.*

Coffee break

11:10–11:30 **F. Fodor**, *On strengthened volume inequalities for* L_p*-zonoids for even isotropic measures.*

11:35–11:55 **T. Jahn**, *How to get rid of symmetry: Orthogonality notions.*

Lunch break

14:00–14:40 **K. Bezdek**, *On non-separable families of positive homothetic convex bodies — the Goodman-Goodman conjecture.*

14:50–15:10 **A. Balitskiy**, *The circle covering theorem by A. W. Goodman and R. E. Goodman revisited.*

Coffee break

15:40–16:00 **N. Frankl**, *Coverings by homothets of a convex body.*

16:05–16:25 **G. Csima**, *Isoptic surfaces in 3D.*

Short Coffee break

16:40–17:00 **A. Zaykov**, *On 1-planar graphs.*

17:05–17:25 **S. Révész**, *On a discretized version of the Rolling Ball Theorem of Blaschke.*

ABSTRACTS
(in alphabetic order by the speaker's surname)

Arseniy Akopyan (IST Austria): *On length of curves passing through boundary points of a planar convex shape*

We study length of curves passing through a fixed number of points at the boundary of a convex shape on a plane. We show that for any convex shape K, there exist four points at the boundary of K such that the length of any curve passing through these points is at least half of perimeter of K. It is also shown that four extreme points that satisfy this condition do not always exist. Moreover, the factor $\frac{1}{2}$ can not be achieved with any fixed number of extreme points. We conclude the paper with few other inequalities related to the perimeter of a convex shape. (Joint work with Vladislav Vysotsky.)

Gergely Ambrus (Renyi Institute, Budapest): *Vector sum estimates in normed spaces*

Consider a set V of n vectors in a d-dimensional real normed space whose norm is at most one. Assume that the vectors sum to 0. We consider several questions. First, what is the best possible bound S so that there always exists an ordering of the elements of V, according to which the initial partial sums have norms bounded by S? This question is due to Steinitz. Second, what is the best bound R so that for any given $k < n$, one choose a subset of V with cardinality k, so that the sum of the elements of this subset has norm at most R? Somewhat surprisingly, there exist such bounds depending only on d, but not on n. We are going to use linear algebraic methods for giving estimates, which are sharp in some cases. (Joint work with I. Barany and V. Grinberg.)

Alexey Balitskiy (Moscow Institute of Physics and Technology): *The circle covering theorem by A. W. Goodman and R. E. Goodman revisited*

In 1945, A. W. Goodman and R. E. Goodman proved the following conjecture by P. Erdős: Given a family of (round) disks of radii $r1, \ldots, r_n$ in the plane it is always possible to cover them by a disk of radius $R = \sum r_i$, provided they cannot be separated into two subfamilies by a straight line disjoint from the disks. Very recently, K. Bezdek and Z. Lángi proved the analogous statement for the homothets of arbitrary centrally-symmetric body, and achieved some success in the non-symmetric case. In the talk I'm going to discuss the argument by A. W. Goodman and R. E. Goodman, and to present the further strengthening of the estimate in the non-symmetric case. (Joint work with Arseniy Akopyan and Mikhail Grigorev.)

Martin Balko (Charles University in Prague): *On the Beer index of convexity and its variants*

Let S be a subset of $\mathbb{R}^d$ with finite positive Lebesgue measure. The Beer index of convexity $b(S)$ of S is the probability that two points of S chosen uniformly independently at random see each other in S. The convexity ratio $c(S)$ of S is the Lebesgue measure of the largest convex subset of S divided by the Lebesgue measure of S. We investigate a relationship between these two natural measures of convexity of S. We show that the Beer index $b(S)$ of every subset S of the plane with simply connected components is at most $\alpha * c(S)$ for some absolute constant α, provided $b(S)$ and $c(S)$ are defined. This implies an affirmative answer to the conjecture of Cabello et al. asserting that this estimate holds for simple polygons. We also consider higher-order generalizations of $b(S)$. For a positive integer k of size at most d, the k-index of convexity of a subset S of $\mathbb{R}^d$ is the probability that the convex hull of a $(k+1)$-tuple of points chosen uniformly independently at random from S is contained in S. We provide almost matching bounds for this parameter and we pose several new open problems. (Joint work with Vít Jelínek, Pavel Valtr, and Bartosz Walczak.)

Imre Bárány (Renyi Institute, Budapest and UCL, London): *Random spherical polytopes in a halfsphere*

A random spherical polytope P_n in a spherically convex set $K \subset \mathbb{S}^d$ is the spherical convex hull of n independent, uniformly distributed random points in K. The (only) interesting case here is when K is a halfsphere. We establish the asymptotic behaviour, as n tends to infinity, of the expectation of several characteristics of P_n, such as facet and vertex number, volume and surface area. (Joint work with Daniel Hug, Matthias Reitzner, and Rolf Schneider.)

Sören Berg (TU Berlin): *Lattice points in centered convex bodies*

In this talk we will discuss upper bounds on the number of lattice points in a convex body having its centroid at the origin. We present an upper bound for arbitrary convex bodies and obtain a sharp upper bound for simplices in particular. These results are continuations of a theorem due to Hermann Minkowski dealing with the (stronger) centrally symmetric assumption. Furthermore, a relation to Ehrharts volume conjecture will be briefly discussed. (Joint work with Martin Henk.)

Károly Bezdek (University of Calgary, University of Pannonia): *On non-separable families of positive homothetic convex bodies — the Goodman-Goodman conjecture*

A finite family $\mathcal{B}$ of balls with respect to an arbitrary norm in $\mathbb{R}^d$ ($d \geq 2$) is called a non-separable family if there is no hyperplane disjoint from $\bigcup \mathcal{B}$ that strictly separates some elements of $\mathcal{B}$ from all the other elements of $\mathcal{B}$ in $\mathbb{R}^d$. In this talk we prove that if $\mathcal{B}$ is a non-separable family of balls of radii $r_1, r_2, \ldots, r_n$ ($n \geq 2$) with respect to an arbitrary norm in $\mathbb{R}^d$ ($d \geq 2$), then $\bigcup \mathcal{B}$ can be covered by a ball of radius $\sum_{i=1}^{n} r_i$. This was conjectured by Erdős for the Euclidean norm and was proved for that case by A. W. Goodman and R. E. Goodman [Amer. Math. Monthly 52 (1945), 494-498]. On the other hand, in the same paper A. W. Goodman and R. E. Goodman conjectured that their theorem extends to arbitrary non-separable finite families of positive homothetic convex bodies in $\mathbb{R}^d$, $d \geq 2$. Besides giving a counterexample to their conjecture, we prove that conjecture under various additional conditions. (Joint work with Zs. Lángi.)

Ted Bisztriczky (University of Calgary/York University): *On neighbourly 4-polytopes*

Because of their definition and extremal property, neighbourly polytopes have been at times regarded as a 'rather freakish' family of polytopes. The definition is counterintuitive and the extremal property is important. Our interest is in the family N of neighbourly 4-polytopes. Then $P \in N$ if any two vertices of P determine a face of P, and the extremal property is the maximum number of faces; that is, if Q is a 4-polytope such that $f_0(Q) \leq f_0(P)$ then $f_k(Q) \leq f_k(P)$ for all k. The rather freakish aspect includes also (for example) that there are approximately 556,000, combinatorially different $P \in N$ with 12 vertices and, except for a couple of construction techniques, we cannot meaningfully describe them. In the talk, we present techniques for visualizing the inner structure of a $P \in N$. Specifically, we describe 'vertex–subpolytope' connections in the case a vertex of P is not contained in a subpolytope of P.

Károly Böröczky (Renyi Institute, ELTE, CEU, Budapest): *Valuations on lattice polytopes*

After the classical results by Ehrhart, Betke & Kneser and McMullen, the theory of valuations on lattice polytopes is witnessing a recent surge parallel to the case of continuous valuations on convex bodies. The talks surveys some exciting new developments.

Balázs Csikós (Eötvös Loránd University, Budapest): *Monotonicity of the perimeter of the convex hull and the area of the intersection of disks in curved planes*

We show that if some disks in the hyperbolic, or Euclidean plane, or in the hemisphere are rearranged so that the distances between their centers do not increase, then the perimeter of their convex hull does not increase, and the area of their intersection does not decrease. We discuss possible generalizations of these statements. These results extend the theorems of R. Alexander, V. N. Sudakov, V. Capoyleas and J. Pach on the monotonicity of the convex hull of some points in the Euclidean plane and a recent result of I. Gorbovickis on the monotonicity of the area of the intersection of disks in the hemisphere, which was proved in the Euclidean plane by K. Bezdek and R. Connelly. (Joint work with Márton Horváth.)

Géza Csima (Budapest University of Technology and Economics): *Isoptic surfaces in 3D*

In our talk we would like to give a short review about the isoptic curves and introduce the generalization of the definition to higher dimensions. We will determine the equation of the isoptic surfaces of rectangles. By using the spherical geometry, we give an algorithm to determine the isoptic surface of convex polyhedra and we will use the Wolfram Mathematica to visualize them.

Ferenc Fodor (University of Szeged / University of Calgary): *On strengthened volume inequalities for L_p-zonoids for even isotropic measures*

We strengthen the volume inequalities for L_p-zonoids of even istropic measures and for their duals due to Ball, Barthe and Lutwak, Yang, Zhang. The $p = \infty$ case yields a stability version of the reverse isoperimetric inequality for centrally symmetric bodies. (Joint work with K. J. Böröczky (Budapest) and D. Hug (Karlsruhe).)

Zoltán Füredi (Renyi Institute, Budapest / University of Illinois Urbana-Champaign): *Subtended Angles*

Suppose that $d \geq 2$ and m are fixed. What is the largest n such that, given any n distinct angles $0 < \theta_1, \theta_2, \dots, \theta_n < \pi$, we can realise all these angles by placing m points in $\mathbb{R}^d$? We say an angle θ is *realised* if there exist points A, B and C such that $A\widehat{B}C = \theta$. E.g., 3 points in general can represent only one prescribed angle, (two obtuse angles cannot be realized by 3 points). Some of our results:

Suppose that $m \geq 5$ and $n \leq 2m - 4$. Then, given any n distinct angles, there is an arrangement of m points in the plane realising of these angles. Moreover, these points may be chosen in convex position.

Here the value of $2m-4$ is the best possible, there exists a set of $2m-3$ (distinct) angles such that no arrangement of m points in any dimension realises all angles in the set.

The results presented are joint work with G. Szigeti and Ballister, Bollobas, Leader, and Walters.

Nóra Frankl (Eötvös Loránd University, Budapest): *Coverings by homothets of a convex body*

Let K be a convex body in $\mathbb{R}^d$ and $\mathcal{F} = \{\lambda_1 K, \lambda_2 K, \dots\}$ a family of its positive, smaller homothets. We define $f(K,K)$ as the infimum of those t for which the following holds: If $\sum_i \lambda_i^d \geq t$, then $\mathcal{F}$ permits a translative covering of K. We improve the former upper bounds on $f(K,K)$, and discuss further results.

Bernardo González Merino (Technische Universität München): *On the missing boundary of the (r,D,R)-diagram*

Santaló in 1961 proved that $r(K) + R(K) \leq D(K)$ holds for any planar convex body K. Here $r(K)$, $R(K)$, and $D(K)$ mean the inradius, circumradius, and diameter of K, respectively, measured w.r.t. the Euclidean ball. Replacing these measures by their relative counterparts $r(K,C)$, $R(K,C)$, and $D(K,C)$ (i.e. measured w.r.t. the convex body C) the inequality keeps valid whenever

$C = -C$. Indeed, Moreno and Schneider in 2012 observed that every diametrically complete set achieves equality in this inequality. If we consider C a non-necessarily centrally symmetric set, we show that diametrically complete sets w.r.t. C do not necessarily fulfill its natural extension

$$s(C)r(K,C) + R(K,C) \leq \frac{s(C)+1}{2} D(K,C)$$

with equality. The number $s(C)$ is the well-known Asymmetry measure of Minkowski, i.e. $s(C) = R(-C,C)$. This observation directly implies the existence of a new inequality relating r, D and R, which we may call the 'Missing boundary of the (r,D,R)-diagram'. (Joint work with René Brandenberg.)

Martin Henk (TU Berlin): *Discrete Slicing Problems*

In analogy to the slicing problem from Convex Geometry we are studying similar problems when the volume functional is replaced by the lattice point enumerator. (Joint work with Artem Zvavitch and Matthew Alexander.)

Matthias Henze (Freie Universität Berlin): *Tight bounds on discrete quantitative Helly numbers*

For a discrete set S of $\mathbb{R}^n$, let $c(S,k)$ be its quantitative Helly number, that is, the smallest number t such that whenever finitely many convex sets have exactly k common points in S, there exist at most t of these sets that already have exactly k common points in S. For $S = \mathbb{Z}^n$, this number was introduced by Aliev et al. [2014] who gave an explicit bound showing that $c(\mathbb{Z}^n,k) = O(k)$ holds for every fixed n. Recently, Chestnut et al. [2015] derived the first sublinear upper bound on $c(\mathbb{Z}^n,k)$, and moreover showed that its growth rate is at least $k^{(n-1)/(n+1)}$. We provide a combinatorial description of $c(S,k)$ in terms of polytopes with vertices in S and use it to improve the previous bounds as follows: (1) We extend the linear bound of Aliev et al. [2014] to general discrete sets S. (2) We close the gap for $\mathbb{Z}^n$ and determine the exact asymptotic behavior of $c(\mathbb{Z}^n,k)$. (Joint work with G. Averkov, B. González Merino, I. Paschke, and S. Weltge.)

Andreas Holmsen (Department of Mathematical Sciences KAIST): *The intersection of a matroid and an oriented matroid*

I will present a 'matroid intersection theorem' where we are intersecting a matroid and an oriented matroid. It can be interpreted as follows: If the positive circuits of the oriented matroid are well-distributed with respect to the matroid, then there is a positive circuit of the oriented matroid whose elements are independent in the matroid. This can be viewed as a combinatorial generalization of the 'colorful Carathéodory theorem' from combinatorial convexity (due to Bárány/Lovász) which has several important applications in discrete geometry.

Thomas Jahn (Technische Universität Chemnitz): *How to get rid of symmetry: Orthogonality notions*

Orthogonality is one of the central concepts in the theory of inner product spaces. Finding an appropriate substitute in normed spaces is therefore a natural task. The most popular orthogonality notions in normed spaces are attributed to Birkhoff and James. In this talk, we discuss their extensions to a generalized Minkowski space, i.e., a finite-dimensional vector space equipped with a gauge.

Katharina Jochemko (Vienna University of Technology): *Unimodality of h^*-vectors for zonotopes and refined Eulerian polynomials: geometry and combinatorics*

The Ehrhart polynomial counts the number of lattice points in integer dilates of a lattice polytope. A central question in Ehrhart theory is to characterize all possible Ehrhart polynomials. An important tool is the h^*-vector of a lattice polytope, which encodes the Ehrhart polynomial in a certain binomial basis. One open question coming from commutative algebra is whether the h^*-vector of an integrally closed lattice polytope is always unimodal. Schepers and Van Langenhoven (2011) proved this for lattice parallelepipeds. Using the interplay of geometry and combinatorics, we generalize their result to zonotopes by interpreting their h^*-vectors in a combinatorial way. More precisely, by taking half-open decompositions we show that the h^*-vector of a zonotope is a weighted sum of certain refined Eulerian polynomials which are related to descent statistics on permutations. From that we see that the h^*-vector of a zonotope is unimodal with peak in the middle and, moreover, that its h^*-polynomial has only real roots. This is joint work with Matthias Beck and Emily McCullough (both San Francisco State University).

Antal Joós (Ybl Faculty of Szent István University, Budapest): *Finding equal-diameter tetrahedralizations of polyhedra*

We show that every simple polyhedron can be tiled into equal-diameter tetrahedra.

Tomasz Kobos (Jagiellonian University in Cracow): *The Banach-Mazur distance in non-asymptotic setting*

The Banach-Mazur distance is a well-established notion of the geometry of Banach spaces originally introduced by Banach. In the context of n-dimensional normed spaces X, Y it measures how far the unit ball of X is from an affine image of the unit ball of Y . In the context of not necessarily centrally-symmetric convex bodies A, B in $\mathbb{R}^n$ it measures how far A is from an affine image of B. The Banach-Mazur distance provides the natural framework for a comparision of the geometry of two convex bodies. It is often used to estimate how much numerical parameters of one space or convex body can differ from the corresponding parameters of the other. The non-symmetric Banach-Mazur compactum is a metric space, which elements are all equivalence classes of affinely equivalent n-dimensional convex bodies with the metric being the logarithm of the Banach-Mazur distance. In the same way we define the symmetric Banach-Mazur compactum, consisting of equivalence classes of affinely equivalent n-dimensional symmetric convex bodies, or equivalently, equivalence classes of isometric n-dimensional Banach spaces. The aim of the talk is to discuss metric properties of both symmetric and non-symmetric Banach-Mazur compactum. As Banach-Mazur distance has proved to be significant in different several different contexts of functional analysis and discrete geometry, these space have been already studied intensively by several specialists in the field of the geometry of Banach spaces. However, most of the obtained results are of the asymptotic character and give no insight in the case of small dimensions. Therefore, our main goal is to explore the metric properties of the Banach-Mazur compacta in the non-asymptotic setting. We shall discuss several possible directions of investigations and present some preliminary results.

István Kovács (Budapest University of Technology and Economics): *Dense point sets with many halving lines*

We construct a dense point set of n points with at least $ne^{\Omega\left(\sqrt{\log n}\right)}$ halving lines. This improves the bound $\Omega(n \log n)$ of Edelsbrunner, Valtr and Welzl from 1997. Our bound is asymptotically the same as the best known lower bound for general point sets. (Joint work with Géza Tóth.)

Flórián Kovács (Budapest University of Technology and Economics): *Some aspects of orthogonal weavings on convex polyhedra*

Let a polyhedral surface be called *weavable* if there exists a set of closed strands of uniform width that provides a complete twofold covering of that surface with orthogonal crossings ('twofold two-way orthogonal weaving'). A necessary but not sufficient condition of weavability is that all vertices of the underlying polyhedron have an angular defect of $n\pi/2, n = 1, 2, 3$ (let such vertices be termed *v-rectangular* for brevity). Weavability of polyhedra is simultaneously investigated by their nets and spherical images (this latter method allows for considering areas of spherical polygons instead of v-rectangular vertices). Because of convexity, the maximum number of v-rectangular vertices of the underlying polyhedron is 8 that restricts our analysis to a set of 301 combinatorially different polyhedra: are there weavable ones within each set? Some of these questions are answered definitely by yes and some others are conjectured to have a negative answer.

Robert Kozma (University of Illinois at Chicago and Budapest University of Technology and Economics): *The best horoball packings for hyperbolic 3- and 4-space*

We will discuss the best know horoball packings of hyperbolic 3- and 4-spaces. We introduce the notion of horoball type, and demonstrate its utility by showing the non-uniqueness result for optimal ball packing configurations of $\mathbb{H}^3$. We proceed to show that it is possible to exceed the conjectured 4-dimensional packing density upper bound due to L. Fejes-Tóth (Regular Figures, 1964). We give several examples of horoball packing configurations that yield higher densities of ≈ 0.71644896 where horoballs are centered at the ideal vertices of certain Coxeter simplex tilings.

Hiu Chung Aaron Lin (London School of Economics and Political Science): *Sets with few ordinary circles*

Let P be a set of n points in the plane. An ordinary circle is a circle (or a line) which goes through exactly three points of P. We show that if P is not contained in a circle or a line, then P spans at least $\frac{3}{8}n^2 - O(n)$ ordinary circles, if n is odd and sufficiently large. This bound is asymptotically tight. (Previously, Hossein Mojarrad and Frank de Zeeuw showed the bound $\frac{1}{4}n^2 - O(n)$ for sufficiently large n, which is tight for even n.) We also describe the extremal and near-extremal examples for each such n.
In order to show this, we prove that if P spans at most Kn^2 ordinary circles, then all but $O(K)$ many points of P lie on a bicircular quartic, a circular cubic, an ellipse, two disjoint circles, or the disjoint union of a circle and a line, if n is sufficiently large depending on K. (Joint work with Konrad Swanepoel, and still in progress.)

Alexander Magazinov (Tel Aviv University): *Half-space depth of the centerline in* $\mathbb{R}^d$

Consider $\mathbb{R}^d$ with a probability measure on it. Given a line ℓ, consider the smallest measure of a half-space containing ℓ. This value is called the half-space depth of ℓ. We prove that for every probability measure μ in $\mathbb{R}^d$ there is a line which has half-space depth at least $1/d + 1/(3d^3)$. This is a slight improvement on the 'trivial' bound $1/d$ given by Rado centerpoint theorem.

Endre Makai, Jr. (Renyi Institute, Budapest): *Densest packings of parallel strings and layers of balls*

Let $L \subset \mathbb{R}^3$ be the union of unit balls, whose centres lie on the z-axis, and are equidistant with distance $2d \in [2, 2\sqrt{2}]$. Then a packing of unit balls in $\mathbb{R}^3$ consisting of translates of L has a density at most $\pi/(3d\sqrt{3-d^2})$, with equality for a certain lattice packing of unit balls. Let $L \subset \mathbb{R}^4$ be the union of unit balls, whose centres lie on the x_3x_4 coordinate plane, and form either a square lattice or a regular triangular lattice, of edge length 2. Then a packing of unit balls in $\mathbb{R}^4$ consisting of translates of L has a density at most $\pi^2/16$, with equality for the densest lattice packing of unit balls in $\mathbb{R}^4$. This is the first class of non-lattice packings of unit balls in $\mathbb{R}^4$, for which this conjectured upper bound for the packing density of balls is proved. Our main tool for the proof is a theorem on (r,R)-systems in $\mathbb{R}^2$. If $R/r \le 2\sqrt{2}$, then the Delone triangulation associated to this (r,R)-system has the following property. The average area of a Delone triangle is at least $\min\{A_0, 2r^2\}$, where A_0 is the infimum of the areas of the non-obtuse Delone triangles. This general theorem has applications also in other problems about packings: namely for $2r^2 \ge A_0$ it is sufficient to deal only with the non-obtuse Delone triangles, which is in general a much easier task. (Joint work with Károly Böröczky and Aladár Heppes.)

Luis Montejano (Universidad Nacional Autonoma De Mexico): *Rotors in triangles and tetrahedra*

A polytope P is circumscribed about a convex body $\phi \subset \mathbb{R}^n$ if $\phi \subset P$ and each facet of P is contained in a support hyperplane of ϕ. We say that a convex body $\phi \subset R^n$ is a *rotor* of a polytope P if for each rotation ρ of $\mathbb{R}^n$ there exist a translation τ so that P is circumscribed about $\tau\rho\phi$. If Q^n is the n-dimensional cube then a convex body Φ is a rotor of Q^n if and only if Φ has constant width. However, there are convex polytopes that have rotors which are not of constant width. We shall give a survey of the main contributions in this fascinating area together with some new results.

Michał Musielak (University of Science and Technology in Bydgoszcz): *Reduced spherical convex bodies*

The aim of this talk is to present some properties of reduced spherical convex bodies on two-dimensional sphere $\mathbb{S}^2$. The spherical convex body R is called to be reduced if its thickness is greater that thickness of any spherical convex body Z such that $Z \subset R$ and $Z \ne R$. In this talk we introduce in particular the theorem which allows to describe the shape of reduced convex bodies of thickness under $\frac{\pi}{2}$ and the result saying that the only reduced convex bodies of thickness at least $\frac{\pi}{2}$ are bodies of constant width.

Deborah Oliveros (Instituto de Matemáticas, Campus Juriquilla, UNAM): *Symmetric Snarks*

In 1976, Loupekine introduced (via Isaacs) a very general way of constructing new snarks from old snarks by cyclically connecting multipoles constructed from smaller snarks. In this talk, we present a new infinite family of snarks, of order $12m$ for each odd $m \ge 3$, which can be embedded with m-fold rotational symmetry. These snarks are constructed similarly to Loupekine's snarks; however, unlike Loupekine's construction, they are constructed beginning with a 3-edge-colorable graph.

Dömötör Pálvölgyi (University of Cambridge): *Polychromatic coloring and cover-decomposition problems in the plane*

Is it true that given a finite point set on a sphere and a set of halfspheres, such that the set system that they induce on the point set is a Sperner family, we can select a subset of the points that meet every halfsphere in at least one but at most two points? I don't know the answer to this question (waiting to be solved by YOU!), but I know that the above holds in the plane if instead of halfspheres we take (pseudo)halfplanes. I will talk about consequences of similar results in polychromatic coloring and cover-decomposition, and also mention several other open problems.

Alexandr Polyanskii (Technion, MIPT, IITP RAS): *On reduced polytopes*

A convex body R in $\mathbb{R}^d$ is called reduced if the minimal width $\Delta(R')$ of each convex body $R' \subset R$ different from R is strictly smaller than the minimal width $\Delta(R)$ of R. We prove some properties of reduced polytopes in $\mathbb{R}^3$. We are going to discuss these results. Our talk is based on the article arXiv:1605.06791

Szilárd Révész (Budapest University of Technology and Economics): *On a discretized version of the Rolling Ball Theorem of Blaschke*

We consider the following question. Suppose we know that two points z and w along a closed convex curve G, being at least of distance t, have necessarily differing normals: i.e. any normal $n(z)$ to G at z and any normal $n(w)$ to G at w differ at least in an angle $a > 0$. What figure can then be outscribed G and passing through a given point z of G? It turns out that we can almost outscribe a regular n-gon with sides at least t and $n = 4k$ such that k exceeds $\frac{\pi}{2a}$. The dual statement and generalizations to higher dimensions are also discussed. In the limit we get back the classical results of Blaschke, as well as the more recent (linearly a.e. version) generalization due to Strantzen.

Laura Silverstein (Technische Universität Wien): *Tensor Valuations on Lattice Polytopes*

A complete classification of symmetric tensor valuations on lattice polytopes that intertwine the special linear group over the integers is established. The scalar-valued case was classified by Betke and Kneser where it was shown that the only such valuations are the coefficients of the Ehrhart polynomial. Extending this result, the coefficients of the discrete moment tensor form a basis for the symmetric tensor valuations.

Jozsef Solymosi (University of British Columbia, Vancouver): *Algebraic Methods in Discrete Geometry*

In this talk we show some applications of the algebraic technique developed by Larry Guth and Netz Katz. We establish new bounds on the number of tangencies and orthogonal intersections determined by an arrangement of curves. First, given a set of n algebraic plane curves, we show that there are $O(n^{3/2})$ points where two or more curves are tangent. In particular, if no three curves are mutually tangent at a common point, then there are $O(n^{3/2})$ curve-curve tangencies. Second, given a family of algebraic plane curves and a set of n curves from this family, we show that either there are $O(n^{3/2})$ points where two or more curves are orthogonal, or the family of curves has certain special properties.

We obtain these bounds by transforming the arrangement of plane curves into an arrangement of space curves so that tangency (or orthogonality) of the original plane curves corresponds to intersection of space curves. We then bound the number of intersections of the corresponding space curves. For the case of curve-curve tangency, we use a polynomial method technique that is reminiscent of Guth and Katz's proof of the joints theorem. For the case of orthogonal curve intersections, we employ a bound of Guth and Zahl to control the number of two-rich points in space curve arrangements. (Joint work with Jordan S. Ellenberg and Josh Zahl.)

Jenő Szirmai (Budapest University of Technology and Economics): *Hyp-hor packings in n-dimensional hyperbolic spaces*

In this talk we deal with the packings derived by horo- and hyperballs (briefly hyp-hor packings) in n-dimensional hyperbolic spaces ($n = 2, 3$) which form a new class of the classical packing problems. We construct in 2- and 3-dimensional hyperbolic spaces hyp-hor packings that are generated by complete Coxeter tilings of degree 1 i.e. the fundamental domains of these tilings are simple frustum orthoschemes and we determine their densest packing configurations and their densities. We prove that in the hyperbolic plane ($n = 2$) the density of the above hyp-hor packings arbitrarily approximate the universal upper bound of the hypercycle or horocycle packing density $\frac{3}{\pi}$ and in 3-dimensional hyperbolic space the optimal configuration belongs to the $[7, 3, 6]$ Coxeter tiling with density 0.83267. Moreover, we study the hyp-hor packings in truncated orthoschemes $[p, 3, 6]$ $(6 < p < 7)$ whose density function is attained its maximum for a parameter which lies in the interval [6.05,6.06] and the densities for parameters lying in this interval are larger that 0.85397. That means that the selocally optimal hyp-hor configurations provide larger densities than the Böröczky-Florian density upper bound 0.85328 for ball and horoball packings but these hyp-hor packing configurations can not be extended to the entirety of hyperbolic space.

István Talata (Ybl Faculty of Szent István University, and University of Dunaújváros): *Examples of convex polytopes with odd translative kissing numbers*

The translative kissing number $H(K)$ of a d-dimensional convex body K is the maximum number of mutually non-overlapping translates of K that can touch K. In previous years, some convex bodies were found in dimensions 3 and higher, whose translative kissing numbers are odd (Joós [2008], Talata [2001, unpublished]). However, none of those examples are polytopes, but they contain some strictly convex arcs on their boundaries. Now, we present explicit examples of convex polytopes in dimensions 3 and higher whose translative kissing numbers are odd.

Viktor Vígh (University of Szeged): *On the angle sum of lines*

What is the maximum of the sum of the pairwise (non-obtuse) angles formed by n lines in the Euclidean 3-space? This question was posed by L. Fejes Tóth in 1959, and it can be naturally extended to higher dimensions as well. L. Fejes Tóth solved the problem in 3-dimensions for n being at most 6, and proved the asymptotic upper bound $n^2 * \pi/5$ (as n tends to infinity). He conjectured that the maximum is asymptotically equal to $n^2 * \pi/6$ (as n tends to infinity). The main result of this talk is an upper bound on the sum of the angles of n lines in the Euclidean 3-space that is asymptotically equal to $n^2 \cdot 3 \cdot \pi/16$ (as n tends to infinity). We are also going to discuss some partial results in higher dimensions. (Joint work with F. Fodor and T. Zarnócz.)

Alexander Zaykov (Moscow Institute of Physics and Technology (MIPT)): *On 1-planar graphs*

A *topological graph* is a graph drawn in the plane with its vertices as distinct points and its edges as Jordan arcs that connect the corresponding points and do not contain any other vertex as an interior point. A *k-planar graph* is such topological graph that each of its edges is crossed at most k times. It is proved that any 1-planar graph on n vertices has at most $4n - 8$ edges and any 2-planar on n vertices has at most $5n - 10$ edges. Therefore, Czap and Hudak conjectured:
Conjecture 1. Let $G = (V, E)$ be a 1-planar graph. Then E can be decomposed into two sets such that one of them induces a planar graph and the other induces a planar forest.
Ackerman proved Conjecture 1 using nontrivial induction. Also, he conjectured:
Conjecture 2. Let $G = (V, E)$ be a 2-planar graph. Then E can be decomposed into three sets such that one of them induces a planar graph and two other sets induce planar forests.
We are going to prove Conjecture 1 using matroids and discuss Conjecture 2. In particular, we proved an estimate on the number of edges in 2-planar graphs without special subgraphs using the approach developed by Pach, Radoicic, Tardos and Toth in their joint work.