# DISCRETE GEOMETRY DAYS$^2$

July 9-12, 2019 Budapest

MŰEGYETEM 1782

**SCHEDULE**

**Venue:** Math. Inst. of the Budapest University of Technology and Economics
*Egry József u. 1., Budapest*, **Building E, Room E1A**

## Tuesday, July 9

**8:45–9:15** *Registration*
**9:15–9:30** *Opening*
**9:30–10:10** **I. Bárány**, *Almost similar triangles [p.5].*
Coffee break
**10:35–11:15** **R. Karasev**, *Gromov's waist of non-radial Gaussian measures and radial non-Gaussian measures [p.9].*
**11:20–12:00** **P. Valtr**, *On Erdős–Szekeres-type problems for k-convex point sets [p.15].*
Lunch break
**14:00–14:40** **N. Mustafa**, *Local Search for Geometric Optimization Problems [p.12].*
**14:45–15:05** **A. Balitskiy**, *Flip cycles in combinatorial configurations [p.4].*
Coffee break
**15:35–15:55** **R. Prosanov**, *The Kuperberg conjecture for translates of convex bodies [p.13].*
**16:00–16:20** **I. Talata**, *On packings of infinite unit cylinders touching a unit ball in the $L^1$-norm [p.15].*
**16:25–16:45** **G. Livshyts**, *On the counterexample to Koldobsky's slicing problem via random rounding [p.11].*
Short break
**17:00–17:20** **T. Jahn**, *Combinatorial aspects of reduced polytopes [p.8].*
**17:25–17:45** **F. Damian**, *On hyperbolic manifolds with uniform discrete involution in the symmetry group [p.6].*
**17:50–18:10** **N. Frankl**, *Bounding the number of unit simplices determined by a set of $n$ points in $\mathbb{R}^d$ [p.7].*

## Wednesday, July 10

**9:00–9:40** **J. Pach**, *Disjointness graphs of strings [p.12].*
**9:45–10:05** **F. Kovács**, *Mechanical complexity of convex polyhedra [p.10].*
Coffee break
**10:35–11:15** **H. Edelsbrunner**, *Shape reconstruction in Bregman geometry [p.7].*
**11:20–12:00** **J. Solymosi**, *The polynomial method in geometry [p.14].*
Lunch break
**14:00–14:40** **R. Pinchasi**, *The book proof of a conjecture of Erdős and Purdy [p.13].*
**14:45–15:05** **A. Polyanskii**, *Perron and Frobenius meet Carathéodory and their other adventures [p.13].*
Coffee break
**15:35–15:55** **E. Makai, Jr.**, *Pairs of convex bodies in planes of constant curvature with axially symmetric intersections of congruent copies [p.11].*
**16:00–16:20** **M. Winter**, *Spectral Methods for Symmetric Polytopes [p.16].*
**16:25–16:45** **Á. Kurusa**, *Finding Needles in a Haystack [p.10].*

| | |
|---|---|
| **16:50–17:10** | **A. Bezdek**, *On the shadows of the cube [p.5].* |
| **19:00–** | *Conference Dinner:* The Kőleves Garden /address: Kazinczy utca 41./ |

## Thursday, July 11

| | |
|---|---|
| **9:00–9:40** | **Z. Patáková**, *Around Radon's theorem [p.13].* |
| **9:45–10:05** | **B. González Merino**, *On discrete and continuous quantitative Helly theorems [p.8].* |
| Coffee break | |
| **10:35–11:15** | **G. Tóth**, *Dense point sets with many halving lines [p.15].* |
| **11:20–11:40** | **D. Korándi**, *Large homogeneous submatrices [p.10].* |
| **11:45–12:05** | **A. Akopyan**, *The Regge symmetry, confocal conics, and the Schläfli formula [p.4].* |
| Lunch break | |
| **14:00–14:40** | **K. Bezdek**, *On uniform contractions of congruent balls [p.5].* |
| **14:45–15:05** | **F. Fodor**, *Random parallelotopes in isotropic vector systems [p.7].* |
| Coffee break | |
| **15:35–15:55** | **S. Avvakumov**, *Convex fair partitions into an arbitrary number of pieces [p.4].* |
| **16:00–16:20** | **G. Ambrus**, *Longest k-monotone chains [p.4].* |
| **16:25–16:45** | **C. Keller**, *The structure of sets of vectors in the plane whose sums span a few directions [p.9].* |
| Short break | |
| **17:00–17:20** | **A. Joós**, *Packing 13 circles in an equilateral triangle [p.8].* |
| **17:25–18:00** | *Open problem session* |

## Friday, July 12

| | |
|---|---|
| **9:00–9:40** | **A. Litvak**, *On the volume ratio between convex bodies [p.11].* |
| **9:45–10:05** | **G. Ivanov**, *Projections of the standard basis and the volume of associated polytopes [p.8].* |
| Coffee break | |
| **10:35–11:15** | **D. Ryabogin**, *On a local version of the fifth Busemann-Petty problem [p.14].* |
| **11:20–11:40** | **V. Yaskin**, *On Grünbaum-type inequalities and their applications [p.16].* |
| **11:45–12:05** | **E. Roldán-Pensado**, *Equipartitions and Mahler's conjecture [p.13].* |
| Lunch break | |
| **14:00–14:20** | **S. Dann**, *Extensions of dual affine quermassintegrals to flag manifolds [p.7].* |
| **14:25–14:45** | **S. Bozóki**, *Polynomial and non-polynomial systems in solving discrete geometry problems [p.5].* |
| **14:50–15:10** | **M. Cossarini**, *Discrete surfaces with length and area and minimal fillings of the circle [p.6].* |
| Coffee break | |
| **15:40–16:00** | **E. Molnár**, *On space form problem [p.12].* |
| **16:05–16:25** | **J. Szirmai**, *Upper bound on density of congruent hyperball packings in hyperbolic* $3-$*space [p.14].* |
| **16:30–16:50** | *Talk cancelled.* |

## ABSTRACTS

**Arseniy Akopyan** (IST Austria): *The Regge symmetry, confocal conics, and the Schläfli formula*

The Regge symmetry is a set of remarkable relations between two tetrahedra whose edge lengths are related in a simple fashion. It was first discovered as a consequence of an asymptotic formula in mathematical physics. Here we give a simple geometric proof of Regge symmetries in Euclidean, spherical, and hyperbolic geometry. (joint work with Ivan Izmestiev)

**Gergely Ambrus** (MTA Renyi Institute): *Longest k-monotone chains*

We study higher order convexity properties of random point sets in the unit square. Given $n$ uniform i.i.d random points, we derive asymptotic estimates for the maximal number of them which are in $k$-monotone position, subject to mild boundary conditions. Besides determining the order of magnitude of the expectation, we also prove strong concentration estimates. We provide a general framework that includes the previously studied cases of $k = 1$ (longest increasing sequences) and $k = 2$ (longest convex chains).

**Sergey Avvakumov** (IST Austria): *Convex fair partitions into an arbitrary number of pieces*

A very natural problem was posed by R. Nandakumar: Given a positive integer $m$ and a convex body $K$ in the plane, cut $K$ into $m$ convex pieces of equal areas and perimeters. In a series of papers by different authors the problem was solved for $m = 2^k$, $m = 3$, and finally for any prime power $m = p^k$. Compared to the previous work on this and similar problems, this time we have found a way to go beyond the usual equivariant (co)homological argument that restricts the possible results to the prime power case. It allowed us to solve the general case of arbitrary $m$. Joint work with A. Akopyan and R. Karasev

**Alexey Balitskiy** (MIT): *Flip cycles in combinatorial configurations*

The 1-skeleton of the celebrated Stasheff associahedron is the graph whose vertices are the triangulations of a convex $n$-gon, and whose edges are the diagonal flips between them. The 2-faces of the associahedron are known to be 4- and 5-gons, which describes all the 'flip relations' in the flip graph. I will introduce the flip graph of Postnikov's planar bicolored graphs (generalizing triangulations) and describe all the flip cycles in this graph, confirming a conjecture of Dylan Thurston from 2004. The proof uses the neat geometry of the cross-sections of zonotopal tilings in 3-space. Joint with Julian Wellman.

**Imre Bárány** (MTA Renyi Inst. and University College London): *Almost similar triangles*

Let $h(n)$ denote the maximum number of triangles in any $n$-element planar set that are *almost regular* meaning that their angles between are 59 and 61 degrees. Our main result is an exact formula for $h(n)$. We also prove $h(n) = n^3/24 + O(n \log n)$ as $n \to \infty$. However, there are triangles $T$ and $n$-point sets $P$ showing that the number of $\epsilon$-similar copies of $T$ in $P$ can exceed $n^3/15$ for any $\epsilon > 0$. This is joint work with Zoltán Füredi.

**András Bezdek** (MTA Rényi Inst. and Auburn University): *On the shadows of the cube*

We say that a polyhedron has the Rupert property if we can make a hole large enough to permit another copy to pass through. It is an old elementary exercise to show that the cube has this property. We show that such hole exists in every direction not parallel to a face of the given cube. In fact we prove that the interior of every perpendicular projection of the cube (shadow) contains a unit square, unless the projection is parallel to one of the faces. Search for the simplest proof for the cube and finding similar results for other polyhedrons lead to a joint work of Antal Joós, Mihály Hujter, Liping Yuan and the presenter.

**Károly Bezdek** (University of Calgary and University of Pannonia): *On uniform contractions of congruent balls*

Let finitely many balls of the same radius be given in a finite dimensional real normed vector space, i.e., in a Minkowski space. Then apply a uniform contraction to the centers of the balls without changing the common radius. Here a uniform contraction is a contraction where all the pairwise distances in the first set of centers are larger than all the pairwise distances in the second set of centers. The talk surveys the results on the monotonicity of the volume of unions (resp., intersections) of congruent balls under uniform contractions of their center points.

**Sándor Bozóki** (MTA SZTAKI - Inst. for Comp. Sci. and Control, Hungarian Acad. of Sci.): *Polynomial and non-polynomial systems in solving discrete geometry problems*

Several problems of geometry can also be written in the form of a system of non-linear equations. Some problems of discrete geometry, such as Littlewood's problem on seven mutually touching infinite cylinders, or Steiner's conic problem lead to polynomial equations. Other problems cannot be transformed into a fully polynomial system, some equations are transcendental. However, through the polynomial approximation of the transcendental terms, an approximating polynomial system can be constructed. Its solutions, found by, e.g., homotopy continuation, are then used as initial points of a Newton's iteration. A numerical example of pie-cutting and the corresponding area-proportional eccentric pie chart is also presented.

**Marcos Cossarini** (Université Paris-Est Marne-la-Vallée): *Discrete surfaces with length and area and minimal fillings of the circle*

We propose to imagine that every Riemannian metric on a surface is discrete at the small scale, made of curves called walls. The length of a curve is its number of crossings with the walls, and the area of the surface is the number of crossings between the walls themselves. We show how to approximate a Riemannian (or self-reverse Finsler) metric by a wallsystem. Wallsystems are dual to square-celled decompositions of the surface. This work is motivated by Gromov's filling area conjecture (FAC) that the hemisphere has minimum area among orientable Riemannian surfaces that fill isometrically a closed curve of given length. (A surface fills its boundary curve isometrically if the distance between each pair of boundary points measured along the surface is not less than the distance measured along the boundary.) We introduce a discrete FAC: every square-celled surface that fills isometrically a $2n$-cycle graph has at least $\frac{n(n-1)}{2}$ squares. This conjecture is equivalent to the FAC extended to surfaces with self-reverse Finsler metric. If the surface is a disk, the discrete FAC follows from Steinitz's algorithm for transforming curves into pseudolines. This gives a new, combinatorial proof that the FAC holds for disks with Riemannian or self-reverse Finsler metric. If time allows, we discuss a newer discretization for directed metrics. In this case the 'fine structure' is a simplicial set, that is (roughly speaking), a triangulation with directed edges. The length of each edge is 1 in one way and 0 in the other way, and the area of the surface is the number of triangles. These discrete surfaces are dual to Postnikov's plabic graphs.

**Florin Damian** (Moldova State University, Inst. of Math. and Comp. Sci.): *On hyperbolic manifolds with uniform discrete involution in the symmetry group*

We discuss methods of discrete geometry in hyperbolic spaces that permit to construct easily new examples of manifolds and to investigate their geometry. However in some cases we can find a manifold with geodesic boundary. If the symmetry group of this boundary of co-dimension one contains an involution without fixed points, then we can complete the construction. We give some examples of hyperbolic manifolds which possess such isometric involution in the present work. The factorization of these manifolds by the above involutions yield complete manifolds whose volume is two times less that the volume of the initial manifolds, for example, for the reconstructed Davis hyperbolic 4-manifold. This investigation leads to an "intermediate" way of representing the hyperbolic manifold by an equidistant polyhedron over a compact basis, as a submanifold of co-dimension one. The geometry of such manifolds in terms of discrete geometry is described too.

**Susanna Dann** (Universidad de los Andes): *Extensions of dual affine quermassintegrals to flag manifolds*

In this talk we discuss generalizations of dual affine quermassintegrals as averages on flag manifolds (where the Grassmannian can be considered as a special case) and extend all known results to this new setting. This is joint work with Grigoris Paouris and Peter Pivovarov.

**Herbert Edelsbrunner** (IST Austria): *Shape reconstruction in Bregman geometry*

How does the measure of dissimilarity affect the reconstruction of shape from point data, and how do we quantify the influence? We approach this question by extending popular Euclidean reconstruction algorithms to Bregman space. A particularly interesting Bregman divergence is the relative entropy, whose infinitesimal version defines the Fisher metric. We explain the connections and illustrate the findings with sample results of the implemented algorithms. Joint work with Katharina Oelsboeck and Hubert Wagner.

**Ferenc Fodor** (University of Szeged): *Random parallelotopes in isotropic vector systems*

Using the expectation of the squared volume of parallelotopes spanned by $d$ independent random vectors distributed according to discrete isotropic measures, we provide a probabilistic approach to the volumetric estimate in the Dvoretzky-Rogers lemma and its subsequent improvement by Pelczyński and Szarek. This talk is based on joint results with Márton Naszódi (Eötvös University, Hungary) and Tamás Zarnócz (University of Szeged, Hungary).

**Nóra Frankl** (London School of Economics): *Bounding the number of unit simplices determined by a set of $n$ points in $\mathbb{R}^d$*

We give general upper bounds on the maximum number of regular unit simplices with $k$ vertices determined by a set $P$ of $n$ points in $\mathbb{R}^d$. For each $d > 3$ we determine the order of magnitude for the smallest $k$ for which the problem is non-trivial. We also study the specific case when $P$ is of diameter one, derive stronger bounds and determine the right order of magnitude in the smallest open case. Joint work with Andrey Kupavskii.

**Bernardo González Merino** (University of Seville): *On discrete and continuous quantitative Helly theorems*

In this talk, we will remember some fundamental theorems of convexity, like Helly, Carathéodory, and Radon theorems. Afterwards, we will discuss some quantitative versions of them, both from the discrete (motivated by a question posed by Aliev, de Loera, and others in 2014) and the continuous (motivated by a question posed by Bárány, Katchalski and Pach in 1982) point of view.

**Grigory Ivanov** (University of Fribourg): *Projections of the standard basis and the volume of associated polytopes*

Let $\{v_1, \ldots, v_n\}$ be the projections of the vectors of the standard basis of $\mathbb{R}^n$ into a $k$-plane $H$. It is not hard to see that there is a linear transformation that maps any small perturbation of $\{v_1, \ldots, v_n\}$ in $H$ into another projection of some orthonormal basis of $\mathbb{R}^n$. We write the first-order approximation formula for such a transformation with the use of exterior algebra. We use it to get first-order necessary conditions in problems of maximization of the volume of a projection of some polytopes. To illustrate the technique, we consider the standard cross-polytope $\lozenge^n = \{x \in \mathbb{R}^n \mid |x_1| + \cdots + |x_n| \leqslant 1\}$, give a geometric meaning of the mentioned conditions for it, and prove that the volume of the projection of $\lozenge^n$ into a 3-plane $H$ is maximal iff $H$ is a coordinate 3-plane.

**Thomas Jahn** (TU Chemnitz): *Combinatorial aspects of reduced polytopes*

A polytope is said to be reduced if its minimum width, i.e., the minimal distance between parallel supporting hyperplanes, is strictly larger than the minimum widths of its polytopal strict subsets. A dimension-independent characterization of reduced polytopes by Lassak involves matchings of vertices and non-incident facets under a metrical constraint. Furthermore, at the first DGD meeting, Sasha Polyanskii presented a combinatorially flavored necessary criterion for the reducedness of polytopes in three-dimensional space. Aiming at the increase of our supply of actual examples of reduced polytopes, we are concerned with two questions in this talk: First, given the dimension, is there a polytope whose vertices cannot be mapped injectively to non-incident facets? Second, given a polytope in three-dimensional space, how many of its vertex-facet matchings violate Polyanskii's criterion?

**Antal Joós** (University of Dunaújváros): *Packing 13 circles in an equilateral* *triangle*

The maximum separation problem is to find the maximum of the minimum pairwise distance of $n$ points in a planar body $\mathcal{B}$ on the Euclidean plane. In this talk this problem will be considered if $\mathcal{B}$ is the equilateral triangle of side length 1 and the number of points is 13. It will be presented the exact separation distance of 13 points in the equilateral triangle of side length 1 and it will be proved a conjecture of Melissen from 1993 and a conjecture of Graham and Lubachevsky from 1995.

**Roman Karasev** (Moscow Inst. of Physics and Tech.): *Gromov's waist of non-radial Gaussian measures and radial non-Gaussian measures*

We study the Gromov waist in the sense of $t$-neighborhoods for measures in the Euclidean space, motivated by the Gromov waist theorem for radially symmetric Gaussian measures. We examine the pancake argument of Gromov and invoke the Caffarelli theorem on monotone transportation to make a better choice of the center point of a pancake. This allows us to simplify the original argument and to prove a natural extension of the waist theorem to not necessarily centrally symmetric Gaussian measures. We also test non-Gaussian radially symmetric measures for the $t$-neighborhood waist property. In some special cases it is possible to establish an analogue of Gromov's theorem. But for a rather wide class of compactly supported radially symmetric measures and their maps into the Euclidean space of dimension at least 2 we show that there is no such $t$-neighborhood waist theorem. This is a joint work with Arseniy Akopyan (IST Austria).

**Chaya Keller** (Technion): *The structure of sets of vectors in the plane whose sums span a few directions*

A classical theorem of Ungar (1982) asserts that any n non-collinear points in the plane determine at least 2[n/2] directions. That is, considering the points as vectors, at least 2[n/2] different directions are spanned by their differences. Jamison and Hill characterized the sets of points for which the minimum is attained. In this talk we discuss the related problem of directions spanned by sums of n vectors in the plane, proposed by Balog and Roche-Newton (2015) in the context of sum-product theorems. Assuming that the vectors are pairwise independent, it is easy to show that the minimal number of directions is 2n-1 if all vectors are contained in the right half-plane, and 2n otherwise. We obtain a structural characterization of all sets of vectors for which the minimum is attained. In particular, we show that for any such set A, almost all elements of A must lie on the union of two quadrics. Joint work with Rom Pinchasi.

**Dániel Korándi** (EPFL): *Large homogeneous submatrices*

A matrix is homogeneous if all of its entries are equal. Let $P$ be a $2 \times 2$ zero-one matrix that is not homogeneous. We prove that if an $n \times n$ zero-one matrix $A$ does not contain $P$ as a submatrix, then $A$ has a $cn \times cn$ homogeneous submatrix for a suitable constant $c > 0$. We further provide an almost complete characterization of the matrices $P$ (missing only finitely many cases) such that forbidding $P$ in $A$ guarantees an $n^{1-o(1)} \times n^{1-o(1)}$ homogeneous submatrix. We apply our results to chordal bipartite graphs, totally balanced matrices, halfplane-arrangements and string graphs. Joint work with János Pach and István Tomon.

**Flórián Kovács** (Budapest Univ. of Tech. and Economics (BME)): *Mechanical complexity of convex polyhedra*

Let $P$ be a convex polyhedron with $f$ faces, $e$ edges and $v$ vertices, and assume it is realized as a homogeneous solid having $S$ stable, $H$ saddle-type and $U$ unstable equilibrium configurations (i.e., standing on a face, edge or vertex over a horizontal surface). Let mechanical complexity $C(P)$ of $P$ be defined as the difference between the sums $f + e + v$ and $S + H + U$ and define mechanical complexity $C(S, U)$ of primary equilibrium classes $(S, U)^E$ as the minimum of mechanical complexities of all polyhedra having $S$ stable and $U$ unstable equilibria. In this talk we show that mechanical complexity in any non-monostatic (i.e., $S > 1$ and $U > 1$) primary equilibrium class $(S, U)^E$ equals the minimum of $2(f+v-S-U)$ over all polyhedra in the same class having simultaneously $f$ faces and $v$ vertices; thus, mechanical complexity of a class $(S, U)^E$ is zero if and only if there exists $P$ such that $f = S$ and $v = U$. We also give both upper and lower bounds for $C(S, U)$ in the cases $S = 1 \neq U$ and $U = 1 \neq S$ (a problem related to a question of Conway and Guy in 1969 asking about the minimal number of faces of convex polyhedra with only one stable equilibrium point), and we offer a complexity-dependent prize for $C(1, 1)$. Joint work with Gábor Domokos, Zsolt Lángi, Krisztina Regős and Péter T. Varga.

**Árpád Kurusa** (Rényi Math Inst. of HAS and Bolyai Math Inst. of U. Szeged): *Finding Needles in a Haystack*

Convex polygons are distinguishable among the convex domains by comparing their visual angle functions on any surrounding circle. This is a consequence of our main result that every segment in a multicurve can be reconstructed if the masking function of the multicurve is known on any surrounding circle.

**Alexander Litvak** (University of Alberta): *On the volume ratio between convex bodies*

In this talk I'll survey known results on the volume ratio between convex bodies.

Cubical and simplex ratios will be discussed as well as the general case and Banach-Mazur type distances.

**Galyna Livshyts** (Georgia Tech): *On the counterexample to Koldobsky's slicing problem via random rounding*

We investigate the volumetric properties of the random polytope which constitutes the counterexample to Koldobsky's slicing conjecture for arbitrary measures. The method allows to obtain sharp dependency on the dimension thanks to an improved version of the standard net argument, called 'random rounding'. This is a joint work with Bo'az Klartag.

**Endre Makai, Jr.** (A. Renyi Inst. of Mathematics, Hungar. Acad. Sci.): *Pairs of convex bodies in planes of constant curvature with axially symmetric intersections of congruent copies*

Let $X$ be $S^2$, $R^2$ or $H^2$. Let $K, L \subsetneqq X$ be closed convex sets with non-empty interiors, and let $\varphi, \psi$ be congruences of $X$. We always suppose $\text{int}\,[(\varphi K) \cap (\psi L)] \neq \emptyset$. A hypercycle domain is a closed convex set bounded by a hypercycle, and a double hypercycle domain is a closed convex set bounded by two hypercycles with common infinite points. (We allow a hypercycle to degenerate to a straight line.) **Theorem 1.** $\forall \varphi, \psi$ we have that $(\varphi K) \cap (\psi L)$ is axially symmetric if and only if (1) for $X = S^2$: $K, L$ are two circles; (2) for $X = R^2$: $K.L$ are (a) two circles, (b) a circle and a parallel strip, (c) a circle and a half-plane, (d) two congruent parallel strips, (e) two half-planes. (3) for $X = H^2$, under the additional hypothesis that the numbers of connected components of $K, L$ are finite: $K, L$ are (a) two circles, (b) a circle and a paracycle (horocycle), (c) a circle and a hypercycle domain, (d) a circle and a double hypercycle domain, (e) two paracycles, (f) a paracycle and a parallel domain of a straight line, (g) two congruent hypercycle domains, (h) two congruent parallel domains of straight lines, (i) a circle of radius $r$ and a closed convex domain bounded by at least two hypercycles, with different base lines, the mutual distances of the hypercycles being at least $2r$. **Theorem 2.** We make the additional hypothesis that $\forall x \in bd\,K$ and $\forall y \in bd\,L$ $K, L$ have supporting circles at $x, y$, for $X = S^2$ of radius less than some number less than $\pi/2$. Then $\forall \varphi, \psi$ we have that $\text{cl}\,\text{conv}\,[(\varphi K) \cup (\psi L)]$ is axially symmetric if and only if $K, L$ are two circles. Joint results with J. Jerónimo-Castro.

**Emil Molnár** (Budapest Univ. of Tech. and Economics (BME)): *On space form problem*

A compact manifold of constant curvature is called space form. This concept can naturally be extended to any space X of the 8 Thurston's (simply connected homogeneous Riemannian) 3-geometry: Thus, we look for a fixed-point-free isometry group G, acting on X (without fixed point) with compact fundamental domain F = X/G, endowed by appropriate face pairing identifications. It turned out that the previous (1984-88) initiative of the author, constructing new hyperbolic space forms (football manifolds)., have got applications in crystallography, e.g. as fullerenes. Nowadays we (István Prok, Jenő Szirmai, Andrei Vesnin) found further space forms (also in other Thurston spaces). Furthermore, other possible applications, as infinite series Cw(2z, 1 < z odd) of nanotubes with z-rotational symmetry are foreseen. Maybe, our experience space in small size can be non-Euclidean as well!!!

**Nabil Mustafa** (ESIEE, Paris): *Local Search for Geometric Optimization Problems*

Local-search is an intuitive approach towards solving combinatorial optimization problems: start with any feasible solution, and try to improve it by local improvements. Like other greedy approaches, it can fail to find the global optimum by getting stuck on a locally optimal solution. In this talk I will present the ideas and techniques behind the use of local-search in the design of provably good approximation algorithms for some combinatorial problems.

**János Pach** (EPFL, Lausanne / Rényi Inst., Budapest): *Disjointness graphs of strings*

Let $\omega(G)$ and $\chi(G)$ denote the clique number and chromatic number of a graph $G$, respectively. The *disjointness graph* of a family of curves (continuous arcs in the plane) is the graph whose vertices correspond to the curves and in which two vertices are joined by an edge if and only if the corresponding curves are disjoint. A curve is called *$x$-monotone* if every vertical line intersects it in at most one point. We solve a 25 years old problem by showing that for arbitrarily large integers $k$, there exist families of $x$-monotone curves such that their disjointness graphs $G$ satisfy $\omega(G) = k$ and $\chi(G) = \Omega(k^4)$. This bound is asymptotically tight. If we drop the condition that the curves are $x$-monotone, then $\chi(G)$ cannot be bounded in terms of $k$. We construct, for every $g > 3$, families of $n$ curves such that the girth of their disjointness graphs $G$ is at least $g$ and $\chi(G) = \Omega_g(\log n)$. This improves a result of Bollobás. Joint work with István Tomon.

**Zuzana Patáková** (IST Austria, Charles University): *Around Radon's theorem*

Radon's theorem is one of the cornerstones of convex geometry. It implies many of the key results in the area such as Helly's theorem and, as very recently shown, also its more robust version, fractional Helly's theorem together with a colorful strengthening of Helly's theorem. Consequently, this yields an existence of weak epsilon nets and a (p,q)-theorem. We show that we can obtain these results even without assuming convexity, replacing it with very weak topological conditions.

**Rom Pinchasi** (Technion, Haifa): *The book proof of a conjecture of Erdős and Purdy*

The following theorem was conjectured by Erdős and Purdy in 1978: Let $P$ be a set of $n > 4$ points in general position in the plane. Suppose that $R$ is a set of points disjoint from $P$ such that every line determined by $P$ passes through a point in $R$. Then $|R| \geq n$. In a recent work with Alexandr Polyanskii we give the book-proof of this conjecture. We will discuss also related results and many interesting open problems.

**Alexandr Polyanskii** (MIPT, CMC ASU, IIPT RAS): *Perron and Frobenius meet Carathéodory and their other adventures*

We are going to discuss how one can apply the Perron-Frobenius Theorem to prove Carathéodory-type results and upper bounds for the size of an almost-equidistant set of diameter 1 (a set of points in $\mathbb{R}^d$ is called *almost-equidistant* if among any three points there are two at distance 1). Our talk is based on two works: arXiv:1901.00540 (joint work with Marton Naszodi) and arXiv:1708.02039. The author is supported by the Russian Foundation for Basic Research, grant No. 18-31-00149 mol a.

**Roman Prosanov** (TU Wien): *The Kuperberg conjecture for translates of convex bodies*

We prove that a convex body admits a dense translative packing if and only if it admits an economical translative covering. This answers positively to the question of W. Kuperberg in the case of translative arrangements.

**Edgardo Roldán-Pensado** (UNAM): *Equipartitions and Mahler's conjecture*

The Mahler volume of a symmetric convex body is the product of its volume and the volume of its dual. There is a conjecture stating the hypercubes have the smallest possible Mahler volume. In 2017, Iriyeh and Shibata published a very long proof in dimension three of this conjecture. I want to discuss a simpler proof in which one of the steps involves solving an equipartition problem.

**Dmitry Ryabogin** (Kent State University, Ohio): *On a local version of the fifth Busemann-Petty problem*

This is a joint work with Maria Alfonseca, Fedor Nazarov and Vlad Yaskin. In

1956, Busemann and Petty posed a series of questions about symmetric convex bodies, of which only the first one has been solved. Their fifth problem asks the following. Let $K$ be an origin symmetric convex body in the $n$-dimensional Euclidean space and let $H_x$ be a hyperplane passing through the origin orthogonal to a unit direction $x$. Consider a hyperplane $G$ parallel to $H_x$ and supporting $K$ and let $C(K,x) = vol(K \cap H_x)dist(0,G)$. If there exists a constant $C$ such that for all directions $x$ we have $C(K,x) = C$, does it follow that $K$ is an ellipsoid? We give the affirmative answers to the problem for bodies in $\mathbb{R}^n$, $n \geq 3$, that are sufficiently close to the Euclidean ball in the Banach-Mazur distance.

**József Solymosi** (University of British Columbia, Vancouver): *The polynomial method in geometry*

The polynomial method for partitioning the Euclidean plane was introduced by Larry Guth and Netz Katz in 2010. Although partitioning techniques were used effectively before that, the Guth-Katz polynomial method led to a series of developments and new results in discrete geometry. In this talk we will review some of the new results and illustrate the advantages and also some limitations of the method.

**Jenő Szirmai** (BME MI. Department of Geometry): *Upper bound on density of congruent hyperball packings in hyperbolic* $3-$*space*

In [Szirmai, J. Decomposition method related to saturated hyperball packings, *Ars Math. Contemp.*, **16** (2019), 349–358.] we proved that to each saturated congruent hyperball packing exists a decomposition of 3-dimensional hyperbolic space $\mathbf{H}^3$ into truncated tetrahedra. Therefore, in order to get a density upper bound for hyperball packings, it is sufficient to determine the density upper bound of hyperball packings in truncated simplices. In this talk we prove, using the above results and the results of papers [Miyamoto, Y. Volumes of hyperbolic manifolds with geodesic boundary, *Topology,* **33/4** (1994), 613–629.] and [Szirmai, J. Hyperball packings in hyperbolic 3-space, *Mat. Vesn.*, **70/3** (2018), 211–221.], that the density upper bound of the saturated congruent hyperball (hypersphere) packings related to the corresponding truncated tetrahedron cells is realized in a regular truncated tetrahedra with density $\approx 0.86338$. Furthermore, we prove that the density of locally optimal congruent hyperball arrangement in regular truncated tetrahedron is not monotonically increasing function of the height (radius) of corresponding optimal hyperball, contrary to the ball (sphere) and horoball (horosphere) packings.

**István Talata** (Ybl Faculty of Szent István University, Budapest): *On packings* *of infinite unit cylinders touching a unit ball in the $L^1$-norm*

W. Kuperberg (1990) posed the following problem: What is the maximum number $n$ of unit radius, infinitely long cylinders with mutually disjoint interiors that can touch a unit ball? Kuperberg conjectured that the answer is 6. The conjecture is still unconfirmed. We consider a generalization of this problem for $d$-dimensional normed spaces, $d \geq 2$. An infinite unit cylinder with an $m$-dimensional axis in a $d$-dimensional normed space is the Minkowski sum $K + S_m$ of the unit ball $K$ of the normed space and an $m$-dimensional affine subspace $S_m$ of $\mathbb{R}^d$. Denote by $c_m(K)$ the maximum number of infinite unit cylinders with an $m$-dimensional axis whose interiors are mutually disjoint and each cylinder touches the unit ball $K$ of the normed space. When $\mathbb{R}^3$ is equipped with the $L^1$-norm, then the unit ball is a 3-dimensional regular octahedron $O_3$. We show $8 \leq c_1(O_3) \leq 9$.

**Géza Tóth** (Rényi Inst. / Budapest Univ. of Tech. and Economics): *Dense point sets with many halving lines*

Suppose that we have a set $P$ of $n$ points in the plane in general position. A line, determined by two points of $P$, is a *halving line* if it has the same number of points of $P$ on both sides. Determining the maximum number of halving lines $f(n)$ of a set of $n$ points turned out to be very important in the analysis of geometric algorithms. We are still very far from the solution, the best bounds are $ne^{c\sqrt{\log n}} \leq f(n) \leq cn^{4/3}$. A planar point set of $n$ points is called $\gamma$-*dense* if the ratio of the largest and smallest distances among the points is at most $\gamma\sqrt{n}$. We construct dense point sets with $ne^{c\sqrt{\log n}}$ halving lines. This improves the bound $cn \log n$ of Edelsbrunner, Valtr and Welzl from 1997. Our construction can be generalized to higher dimensions. Joint work with István Kovács.

**Pavel Valtr** (Charles University, Prague): *On Erdős–Szekeres-type problems for $k$-convex point sets*

We study Erdős–Szekeres-type problems for $k$-convex point sets, a recently introduced notion that naturally extends the concept of convex position. A finite set $S$ of $n$ points is *$k$-convex* if there exists a spanning simple polygonization of $S$ such that the intersection of any straight line with its interior consists of at most $k$ connected components. We address several open problems about $k$-convex point sets. In particular, we extend the well-known Erdős–Szekeres Theorem by showing that, for every fixed $k \in \mathbb{N}$, every set of $n$ points in the plane in *general position* (with no three collinear points) contains a $k$-convex subset of size at least $\Omega(\log^k n)$. We also show that there are arbitrarily large 3-convex sets of $n$ points in the plane in general position whose largest 1-convex subset has size $O(\log n)$.

This gives a solution to a problem posed by Aichholzer et al. (2014). We prove that there is a constant $c > 0$ such that, for every $n \in \mathbb{N}$, there is a set $S$ of $n$ points in the plane in general position such that every 2-convex polygon spanned by at least $c \cdot \log n$ points from $S$ contains a point of $S$ in its interior. This matches an earlier upper bound by Aichholzer et al. (2014) up to a multiplicative constant and answers another of their open problems. Joint work with M. Balko, S. Bhore, and L. Martínez-Sandoval.

**Martin Winter** (TU Chemnitz): *Spectral Methods for Symmetric Polytopes*

The study of eigenspaces of symmetric graphs seems to give an approach to the following questions: Is a sufficiently symmetric polytope already uniquely determined by its edge graph? Furthermore, does such a polytope realizes all the combinatorial symmetries of its edge graph? This can shown to be true for a certain strong symmetry requirement: distance-transitivity. The question is whether the same techniques can extend the result to a larger class of symmetric polytopes.

**Vladyslav Yaskin** (University of Alberta): *On Grünbaum-type inequalities and their applications*

Let $K$ be a convex body in $\mathbb{R}^n$. According to an old result of Grünbaum, if $K$ is cut by a hyperplane passing through its centroid, then the volumes of the two resulting pieces cannot be too small (they are larger than $1/e$ times the volume of $K$). We will review recent generalizations of Grünbaum's result for projections and sections of convex bodies. As an application of these techniques, we will obtain an extension of a result of Fradelizi about maximal sections to the case of intrinsic volumes.